

Quantitative Aptitude

TIME AND WORK

Performing or doing work of any amount involves efforts of person (s) Over a period of time. Therefore, the number of persons (P), the quantity of work (W) and the period of time taken (T) are important variables in problems related to Time and Work'. Moreover time (T) taken to do a work depends not only on how many persons are employed to do it but also on how efficient they are. Efficiency here means *rate of doing same work*. This aspect comes into picture when the problem involves comparison of works done by different categories of persons. For instance, efficiencies of man, woman, boy, girl in general are different. Even efficiency of one man may not be same as that of other; but unless otherwise specifically stated in the problem, all men or women working in a group are assumed to do work with equal efficiency.

The problems on Time and Work can be solved by following two methods :

(i) Ratio and Proportion Method : Since the problem concerning to Time and Work have proportional relation, these can be solved by this method also. We have tried to solve questions based on Time and Work in the chapter mentioned.

(ii) Unitary Method : This is a very simple and useful method. The term 'unitary' is self-evident. In this method we first proceed to reduce the problem to either work done by one person or work done in 1 day and so on as per the requirement of the problem. Let us try to understand it.

Let A can complete a work in 2 days. So, on day 1, A will do half of the work and on day 2 he will do the remaining half of the work. Now, suppose B too alone can do a piece of work in 2 days. So B will do half of the work on 1st day and half of the work on 2nd day. Now, let A and B start together. On day 1 A will do half of the work and B also does half of the work. So, half work + half work = one work is completed by A and B on the day 1 itself. Mathematically,

$$A's \text{ one day's work} = \frac{1}{2}$$

$$B's \text{ one day's work} = \frac{1}{2}$$

$$(A + B)'s \text{ one day's work} = \frac{1}{2} + \frac{1}{2} = 1$$

Following simple facts must be kept in mind while solving problems on Time and Work.

1. The efficiency of workmen remains same while working. In other words, each workman does the same work each day.

2. The total work is assumed to be 1.

3. If a man can do a piece of work in 20 days, then

it implies that he will do $\frac{1}{20}$ th of the work in 1 day. And

conversely if a man can do $\frac{1}{20}$ of the work in 1 day, he will do the work in 20 days.

4. One day's work

$$\frac{1}{\text{Number of days required to complete the work}}$$

5. Obviously, number of days required to complete the work

$$\frac{1}{\text{One day's work}}$$

6. If the number of men engaged to do a piece of work be changed in the ratio $m:n$ the time required for the work will be changed in the inverse ratio $n:m$.

7. If A is x times as good a work man as B, then A

will take $\frac{1}{x}$ of time that B takes to do a certain work. In other words, if A's work : B's work = 2:1, then ratio of the time taken by A and B to finish the work = 1:2,

i.e., A will take $\frac{1}{x}$ of the time taken by B to finish that work.

8. From the above facts we may derive that work, workman and time bear the following relations :

(a) More men, less days and conversely more days, less men. *(Indirect proportion)*

(b) More men, more work and conversely more work, more men. *(Direct proportion)*

(c) More days, more work and conversely more work, more days . *(Direct proportion)*

Problems on Time and Work can be of following types :

(i) Work, labourer and time

(ii) Work, wage and time.

(iii) Pipe, cistern and time

SOLVED EXAMPLES

1. If 8 men and 12 boys can complete a piece of work in 12 days, in what time will 40 men and 45 boys complete another piece of work three times as great, supposing 16 men can do as much work in 8 hours as 12 boys can do in 24 hours?

(SSC Graduate Level (UDC) Main Exam. 2003)

Sol. : 16 men \Rightarrow 8 hours

$$\Rightarrow 16 \times 8 \text{ men} \Rightarrow 1 \text{ hour}$$

Similarly, 12 boys \Rightarrow 24 hours

$$\Rightarrow 12 \times 24 \text{ boys} \Rightarrow 1 \text{ hour}$$

$$\therefore 16 \times 8 \text{ men} = 12 \times 24 \text{ boys} \Rightarrow 4 \text{ men} = 9 \text{ boys}$$

$$\therefore 8 \text{ men} + 12 \text{ boys} = 9 \times 2 \text{ boys} + 12 \text{ boys} = 30 \text{ boys}$$

$$40 \text{ men} + 45 \text{ boys} = 10 \times 9 \text{ boys} + 45 \text{ boys}$$

$$= 135 \text{ boys}$$

Now,

\because 30 boys complete the work in 12 days.

\therefore 1 boy can complete 1 work in 30×12 days.

$$\therefore 135 \text{ boys complete 1 work in } = \frac{30 \times 12}{135} \text{ days}$$

$$\therefore 135 \text{ boys can complete 3 works in } = \frac{30 \times 12 \times 3}{135}$$

$$= 8 \text{ days}$$

2. A and B can do a work in 10 days, B and C can do the same work in 20 days, while C and A can do it in 15 days. In how many days can C alone do the same work? (SSC Matric Level (LDC) Main Exam, 2003)

Sol. : $(A + B)$'s 1 day's work $= \frac{1}{10}$

$$(B + C)'s 1 \text{ day's work} = \frac{1}{20}$$

$$(C + A)'s 1 \text{ day's work} = \frac{1}{15}$$

On adding,

$$2(A + B + C)'s 1 \text{ day's work} = \frac{1}{10} + \frac{1}{20} + \frac{1}{15}$$

$$= \frac{6+3+4}{60} = \frac{13}{60}$$

$$\Rightarrow (A + B + C)'s 1 \text{ day's work} = \frac{13}{120}$$

$$\text{Now, } C's 1 \text{ day's work} = (A + B + C)'s 1 \text{ day's work} - (A + B)'s 1 \text{ day's work}$$

$$= \frac{13}{120} - \frac{1}{10} = \frac{13-12}{120} = \frac{1}{120}$$

Hence, C alone can do the work in 120 days.

3. A can do a piece of work in 10 days and B in 20 days. They work together but 2 days before the completion of the work, A leaves. In how many days was the work completed?

(SSC Graduate level (Assistant Grade) Main Exam, 2004)

Sol. : A leaves 2 days before the completion of work.

$$\text{Now, } B's 1 \text{ day's work} = \frac{1}{20}$$

$$\therefore B's 2 \text{ days' work} = 2 \times \frac{1}{20} = \frac{1}{10}$$

$$\text{Remaining work} = 1 - \frac{1}{10} = \frac{9}{10}$$

Again,

$$A's 1 \text{ day's work} = \frac{1}{10}$$

$$B's 1 \text{ day's work} = \frac{1}{20}$$

$$\therefore (A + B)'s 1 \text{ day's work} = \frac{1}{10} + \frac{1}{20} = \frac{2+1}{20} = \frac{3}{20}$$

Now, $\frac{3}{20}$ work is done by A and B in 1 day.

$$\therefore \frac{9}{10} \text{ work is done by A and B in } = \frac{20}{3} \times \frac{9}{10} \\ = 6 \text{ days.}$$

$$\therefore \text{Total time taken} = 2 + 6 = 8 \text{ days.}$$

4. A and B could do a piece of work in 30 days. After working for 10 days, they are assisted by C and the work is finished in 10 days. If C does as much work in 2 days as B does in 3 days, in how many days could A do the same work alone?

(SSC Graduate level (UDC) Main Exam, 2004)

$$\text{Sol. } (A + B)'s 1 \text{ day's work} = \frac{1}{30} \quad \text{.....(i)}$$

$$(A + B)'s 10 \text{ days' work} = 10 \times \frac{1}{30} = \frac{1}{3}$$

$$\text{Remaining work} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\text{Now, } \therefore (A + B + C)'s 10 \text{ days' work} = \frac{2}{3}$$

$\therefore (A + B + C)'s 1 \text{ day's work}$

$$= \frac{2}{3 \times 10} = \frac{1}{15} \quad \text{.....(ii)}$$

\therefore From equation (ii) - (i)

C's 1 day's work

$$= \frac{1}{15} - \frac{1}{30} = \frac{2-1}{30} = \frac{1}{30}$$

\therefore C alone can finish the work in 30 days.

Now, it is given that C does as much work in 2 days as B does in 3 days.

\therefore The work which C does in 30 days, will be done by

$$B \text{ in } = 30 \times \frac{3}{2} = 45 \text{ days.}$$

\therefore B alone will finish the work in 45 days.

$$\therefore B's \text{ 1 day's work} = \frac{1}{45} \quad \text{(iii)}$$

\therefore From equation (i) - (iii) A's 1 day's work

$$= \frac{1}{30} - \frac{1}{45} = \frac{3-2}{90} = \frac{1}{90}$$

\therefore A alone can finish the work in 90 days.

5. A is twice as good a workman as B and together they can complete a piece of work in 14 days. In how many days can it be done by A alone?

(SSC Income Tax & Central Excise
(Tax Assistant) Exam, 2004)

Sol. : Let the time taken by B alone to complete the work be $2x$ days. Since A is twice as good a workman as B, then time taken by A alone to complete the work = x days. Now,

$$\text{Work done by A in 1 day} = \frac{1}{x}$$

$$\text{Work done by B in 1 day} = \frac{1}{2x}$$

$$\text{Work done by (A + B) in 1 day} = \frac{1}{x} + \frac{1}{2x} = \frac{2+1}{2x} = \frac{3}{2x}$$

$$\text{Clearly, } \frac{3}{2x} = \frac{1}{14} \Rightarrow 2x = 42$$

$$\therefore x = \frac{42}{2} = 21$$

Hence, A alone can complete the work in 21 days.

6. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

$$(1) 12\frac{1}{2} \text{ days} \quad (2) 13\frac{1}{2} \text{ days}$$

$$(3) 14\frac{1}{2} \text{ days} \quad (4) \text{None of these}$$

Sol. : (1) Let 1 man's 1 day's work = x & 1 boy's 1 day's work = y .

$$\text{Then, } 2x + 3y = \frac{1}{10} \text{ and } 3x + 2y = \frac{1}{8}$$

Solving, we get

$$x = \frac{7}{200} \text{ and } y = \frac{1}{100}$$

\therefore (2 men + 1 boy)'s 1 day's work

$$= \left(2 \times \frac{7}{200} + 1 \times \frac{1}{100} \right) = \frac{16}{200} = \frac{2}{25}$$

So, 2 men and 1 boy together can finish the work in

$$\frac{25}{2} = 12\frac{1}{2} \text{ days}$$

Hence, option (1) is correct.

7. Two pipes A and B can fill a tank in 15 hours and 20 hours respectively, while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning. After 10 hours C is closed. In how much time will the tank be full?

$$(1) 13 \text{ hours} \quad (2) 12 \text{ hours}$$

$$(3) 12\frac{1}{2} \text{ days} \quad (4) 15 \text{ hours}$$

Sol. (2) Part of the tank filled by pipe A in 1 hour = $\frac{1}{15}$

Part of the tank filled by pipe B in 1 hour = $\frac{1}{20}$

Part of the tank emptied by pipe C in 1 hour = $\frac{1}{25}$

\therefore Part of the tank filled in 10 hours if all three pipes are opened simultaneously

$$= 10 \left(\frac{1}{15} + \frac{1}{20} - \frac{1}{25} \right)$$

$$= 10 \left(\frac{20+15-12}{300} \right) = \frac{23}{30}$$

$$\text{Remaining Part} = 1 - \frac{23}{30} = \frac{30-23}{30} = \frac{7}{30}$$

Part of the tank filled by pipes A and B in 1 hour

$$= \frac{1}{15} + \frac{1}{20} = \frac{4+3}{60} = \frac{7}{60}$$

\therefore Time taken to fill $\frac{7}{30}$ part of the tank

$$= \frac{60}{7} \times \frac{7}{30} = 2 \text{ hours}$$

Hence, the tank will be filled in $10 + 2 = 12$ hours.

8. A man and a woman, working together, can complete a piece of work in 33 days. The ratio of their efficiencies of working is 14 : 11. In how many days will a woman, working alone, complete the work?

$$(1) 70 \text{ days} \quad (2) 72 \text{ days}$$

$$(3) 75 \text{ days} \quad (4) \text{None of these}$$

Sol. (3) Ratio of the efficiencies of man and woman

$$= 14 : 11$$

\therefore Ratio of time taken by them respectively = 11 : 14

Let time taken by the man alone to complete the work = $11x$ days and time taken by the woman = $14x$ days

$$\text{Men's 1 day's work} = \frac{1}{11x}$$

$$\text{Woman's 1 day's work} = \frac{1}{14x}$$

According to the question,

$$\frac{1}{11x} + \frac{1}{14x} = \frac{1}{33} \Rightarrow \frac{14+11}{154x} = \frac{1}{33}$$

$$\Rightarrow \frac{25}{154x} = \frac{1}{33}$$

$$\Rightarrow 154x = 33 \times 25 \Rightarrow x = \frac{33 \times 25}{154}$$

\therefore Time taken by the woman to complete the work

$$= 14x = \frac{14 \times 33 \times 25}{154} = 75 \text{ days}$$

9. Working 5 hours a day, A can complete a work in 8 days and working 6 hours a day, B can complete the same work in 10 days. Working 8 hours a day, they can jointly complete the work in

- (1) 3 days (2) 4 days
 (3) 4.5 days (4) 5.4 days

(SSC Combined Graduate Level PT Exam,
 27.07.2008 First Sitting)

- Sol. (1) A Working 5 hours a day, A can complete a work in 8 days.

i.e. A can complete the work in 40 hours.

Similarly,

B will complete the same work in 60 hours.

$\therefore (A + B)$'s 1 hour's work

$$= \frac{1}{40} + \frac{1}{60} = \frac{3+2}{120} = \frac{5}{120} = \frac{1}{24}$$

Hence, A and B together will complete the work in 24 hours.

\therefore They can complete the work in 3 days working 8 hours a day.

10. An empty tank can be filled by pipe A in 4 hours and by pipe B in 6 hours. If the two pipes are opened for 1 hour each alternately with first opening pipe A, then the tank will be filled in

- (1) $1\frac{3}{4}$ hours (2) $2\frac{3}{5}$ hours
 (3) $4\frac{2}{3}$ hours (4) $5\frac{1}{2}$ hours

(SSC Combined Graduate Level PT Exam,
 27.07.2008 Second Sitting)

- Sol. (3) Part of the tank filled in first 2 hours

$$= \frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12} \text{ Parts}$$

\therefore Part of the tank filled in first 4 hours

$$= \frac{2 \times 5}{12} \text{ parts} = \frac{5}{6} \text{ parts}$$

$$\text{Remaining part} = 1 - \frac{5}{6} = \frac{1}{6}$$

Now it is the turn of pipe A

$$\text{Time taken to fill } \frac{1}{4} \text{ part} = 1 \text{ hour}$$

$$\therefore \text{Time taken to fill } \frac{1}{6} \text{ part} = \frac{1}{6} \times 4 = \frac{2}{3} \text{ hour}$$

$$\therefore \text{Total time} = 4 + \frac{2}{3} = 4\frac{2}{3} \text{ hours}$$

EVALUATE YOURSELF

SUBJECTIVE TYPE QUESTIONS

- A and B together can do a piece of work in 7 days. If AD alone does twice as much work as B in a given time, find how long A alone would take to do the work?
- A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would all take to finish the work together ?
- A, B and C can do a piece of work in 24, 30 and 40 days respectively. They Start to work together but C leaves 4 days before the completion of the work. In how many days is the work done ?
- A can do as much work in 2 days as B in 3 days and B as much in 4 days as C in 5 days. In what time could A, B and C together do a piece of work when A can do it in 11 days.
- 25 men would finish a piece of work in 120 days. But at the end of every 10 days, 10 additional men are employed. In how many days will it be finished?
- Two men and three boys can do a piece of work in 8 days and three men and two boys can do it in 7 days. If the daily wages of a boy be Rs. 9 what will be the weekly wages of a man?
- A can do a piece of work in 36 days, B in 54 days and C in 72 days. All the three began the work together but A left after 8 days and B, 12 days before the completion of the work. How many days in all did C put in till the entire work was finished?
- If two men or three women can do a work in 28 days, in how many days will 1 man and 9 women can do that work ?
- A can do a piece of work in 90 days, B in 40 days and C in 12 days. They work for a day each in turn i.e. first day A does it alone, B does it the second day and C the third day. After that A does it for another day and soon after finishing the work they get Rs. 240. If the wages are divided in proportional to the work done by them, find what will each get?
- A takes twice as long as B and C together take to do a piece of work and B takes twice as long as A and C together. However three men together can complete the work in 30 days. How long would C alone take to do the work ?
- A can do a piece of work in 25 days which B alone can do in 20 days. A started the work and was joined by B after 10 days. Find the time taken to finish the work.
- Three labourers A, B, C were given a contract of Rs. 750 for doing a certain piece of work. All the three together can finish this work in 8 days. A and C together can do it in 12 days, while A and B together can do it in 13 — days. How the money will be divided among them?
- 25 men are employed to do a piece of work which they could finish in 20 days, but the men drop off by 5 at the end of every 10 days. In what time will the work be completed ?

- 14.** A and B undertake to do a piece of work for Rs. 37.50. A alone could do it in 20 days and B in 30 days. With the assistance of C, they finished it in 8 days. How should the money be divided?

15. A and B can do a piece of work in 10 days, B and C in 15 days and A and C in 25 days; they all work at it together for 4 days; A then leaves and B and C go on together for 5 days more and then B leaves; in how many more days will C complete the work?

OBJECTTVE TYPE QUESTTIONS

ANSWERS

1. $10\frac{1}{2}$ days 2. 10 days
 3. 11 days 4. 5 days
 5. 60 days 6. Rs. 126
 7. 24 days 8. 8 days
 9. A = Rs. 24, B = Rs. 54, C = Rs. 162.
 10. 90 days 11. $16\frac{2}{3}$ days
 12. Rs. 200, Rs. 250, Rs. 300
 13. $23\frac{1}{3}$ days
 14. A = Rs. 15, B = Rs. 10, C = Rs. 12.50.
 15. 76
 16. (1) 17. (3) 18. (2) 19. (4) 20. (2)
 21. (3) 22. (2) 23. (2) 24. (2) 25. (2)
 26. (3) 27. (4) 28. (1) 29. (4) 30. (3)